CRITERIA FOR EXCITATION OF A SHOCK WAVE AND ITS INTENSITY DURING AN EXPLOSION IN A RAREFIED GAS

Yu. A. Medvedev and V. D. Khokhlov

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The effect of the initial pressure of the surrounding gas on the intensity of the shock wave (SW) formed during the dispersion of material vaporized by a powerful laser pulse is examined. The initial stage of expansion of the plasma generated through the focusing of powerful laser radiation on the surface of a solid material in air was studied experimentally in [1, 2]. The times of formation and the initial radii of the SW were recorded on the photoscans of the SW front radiation presented in these reports. It is found that at an air pressure below ~0.1 mm Hg the recordings of the intrinsic radiation of a flare do not differ from the corresponding recordings in a vacuum. For instance, in [2] a bright shock front was observed at a pressure of 0.18 mm Hg, while at a pressure of 0.1 mm Hg SW radiation was not detected. In [2] the hypothesis was made that at an air pressure below ~0.1 mm Hg a SW is not formed and the interaction of the vaporized material with the surrounding gas has a diffusion nature. However, in [1] SW were detected by the Schlieren method at a considerably lower pressure, about $2 \cdot 10^{-2}$ mm Hg. It will be shown below that the sharp decrease observed in the brightness of the radiation of SW fronts generated during laser heating of a solid material in a rarefied gas is explained by the rapid decrease in the maximum SW velocity at a pressure below ~0.1 mm Hg. The expansion of the vaporized material at a pressure of the surrounding gas much less than 0.1 mm Hg is also examined.

Criterion for Formation of a Weakly Radiating Shock Wave. Let us determine the pressure p_* of the surrounding gas below which the initial velocity of the SW produced during vaporization of a solid material by a powerful pulse of laser radiation begins to decrease. We will assume that the vaporized material is surrounded by a weightless opaque shell having the same properties as the gas behind the forming SW front, while the increases in mass, momentum, and energy of the gas in the space occupied by the fronts of cylindrical and spherical SW per unit area of the front surface are equal to the corresponding values for a plane SW.

Under these assumptions the equation of continuity can be written in the following form [3]:

$$\frac{R\rho_1}{k} = \int_{-0.5L}^{0.5L} [\rho(x) - \rho_1]\, dx$$

from which

$$R = (\rho_2/\rho_1 - 1)\, Lk/2 \tag{1}$$

where R is the initial SW radius (the distance travelled by the shell by the moment of SW formation), $\rho(x)$ is the distribution of gas density in the SW front, ρ_2/ρ_1 is the ratio of gas density behind the SW front to the density of the undisturbed gas, L is the SW front thickness, and k = 1, 2, and 3 for plane, cylindrical, and spherical symmetry, respectively. It can be confirmed that Eq. (1) is valid for the three different $\rho(x)$ distributions presented in [4] (p. 154). This indicates the weak dependence of R on $\rho(x)$.

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For strong SW in air $L \approx 1.4 l$ [5], consequently

$$R \approx 0.7\ (\rho_2/\rho_1 - 1)\ l \tag{2}$$

where l is the mean free path length of molecules in the undisturbed gas.

In the experiments of [1] the velocity of dispersion of the vapors at the initial times was $5 \cdot 10^5$-$5 \cdot 10^6$ cm/sec. At such velocities and an air pressure of 1 mm Hg the degree of compression in the SW is 11-15 [6]. Taking $\rho_2/\rho_1 = 13$ and $k = 3$ we find from Eq. (2) that $R \approx 1.2$ mm. The initial SW radius determined from the recording of [1] is ~ 1 mm.

We can determine the pressure p_* from the equation of conservation of energy. For this we must find the energy W imparted by the vaporized material to the surrounding gas up to the moment of SW formation. Using the function $\rho(x)$ obtained in [5] for the kinetic energy of the gas in a SW front propagating through a stationary gas we find

$$W_1/S = C\ (\rho_2/\rho_1)\ D^2\rho_1 L/2$$

$$C\left(\frac{\rho_2}{\rho_1}\right) = \frac{1}{2}\left(\frac{\rho_2}{\rho_1} - 1\right) + \frac{\rho_1}{\rho_2} + \frac{1}{4}\frac{\rho_1}{\rho_2}\ln\frac{\rho_2/\rho_1 + \exp(-2)}{\rho_2/\rho_1 + \exp(2)} - \frac{1}{4}\ln\frac{1 + (\rho_2/\rho_1)\exp(2)}{1 + (\rho_2/\rho_1)\exp(-2)} \tag{3}$$

where S is the area of the SW front surface. It follows from the equations at the SW front ([7], p. 57) that the increase in the internal energy of the gas in a powerful SW front, including the energy of excitation of the internal degrees of freedom of the molecules, is approximately equal to W_1. Thus, for powerful SW in air

$$W/S \approx 1.4C\ (\rho_2/\rho_1)\ D^2\rho_1 l \sim D^2 m/Q$$

where m is the mass of a molecule and Q is its gas kinetic cross section.

The maximum initial SW velocity is approximately equal to the asymptotic velocity U of dispersion of the vapors into a vacuum, which is reached when the gas dynamic pressure is close to the pressure generated in a powerful explosion. In the initial stage of a powerful explosion the average pressure in the inner region of the explosion is $\sim p_2/2$ ([7], p. 88), where p_2 is the gas pressure behind the SW front. Consequently, the internal energy of the vapors at the moment of SW formation is

$$E_1 \approx SR\rho_1 U^2/(\gamma + 1)\ (\gamma_1 - 1)\ k$$

where γ and γ_1 are the ratios of the specific heat capacities of the surrounding gas and the vaporized material, respectively. The kinetic energy W_2 of the vapors at this stage is 2-4 times greater than E_1 [1]. Taking this into account ($W_2 = 3E_1$) the equation of conservation of energy takes the form

$$4S\ (p_*)\ R\ (p_*)\ \rho_1\ (p_*)\ U^2/k\,(\gamma + 1)\ (\gamma_1 - 1) + 1.4C\ (\rho_2/\rho_1)\ S\ (p_*)\ \rho_1 l = E \tag{4}$$

where E is the liberated energy. Since during laser heating of a massive target the dispersing material is approximately bounded by a hemisphere, we obtain from (2) and (4)

$$p_* \approx 6.2\left(\frac{\rho_2}{\rho_1} - 1\right) BU \sqrt{\frac{A}{E}}\left[\frac{2\,(\rho_2/\rho_1 - 1)}{(\gamma + 1)(\gamma_1 - 1)} + C\left(\frac{\rho_2}{\rho_1}\right)\right]^{1/2} \tag{5}$$

$$(A = \rho_1 l,\ B = p_1 l)$$

where for air $A \approx 7.8 \cdot 10^{-8}$ kg/m^2 and $B \approx 6.4 \cdot 10^{-3}$ N/m. Since $U \sim E^n$ where $n = 0.15$-0.3 [1, 8], the pressure p_* depends weakly on E.

Under the conditions of the experiments of [1] $E \approx 3$ J and $U \approx 1.4 \cdot 10^7$ cm/sec. Then setting $\rho_2/\rho_1 = 9$ and $\gamma_1 = 5/3$ in (5) we obtain $p_* \approx 0.2$ mm Hg, which is close to the air pressure (~ 0.1 mm Hg) below which a sharp decrease was observed in the brightness of the air radiation behind the SW front in [1, 2]. For the given example $R \approx 4$ mm and the process of SW formation is completed after the end of the laser pulse.

Assuming that when p_1 does not differ very much from p_* a powerful SW is formed and the motion after the end of the laser pulse is adiabatic, we can find the dependence of the initial SW velocity D on p_1 when $p_1 < p_*$. The initial SW radius is $R \sim l \sim p_1^{-1}$, the average density and average pressure in the vaporized material at the moment of SW formation are

$$\rho_3 \sim R^{-3} \sim p_1^3, \qquad p_3 \sim \rho_3^{\gamma_1} \sim p_1^{3\gamma_1}$$

respectively, the gas pressure behind the SW front is

$$p_2 \approx 2\,\rho_1 D^2/(\gamma+1) \approx 2p_3 \sim p_1^{3\gamma_1}$$

and the initial SW velocity is

$$D \sim p_1^{(3\gamma_1-1)/2}$$

Consequently, powerful SW are formed when

$$p_1 \gtrsim p_*\,(10c_1/U)^{2/(3\gamma_1-1)}$$

where c_1 is the speed of sound in the undisturbed gas. A decrease in the velocity of the forming SW together with a decrease in the gas density behind the SW front when $p_1 \lessdot p_*$ leads to the abrupt dimming of the brightness of the radiation of the gas behind the SW front which was detected in [1, 2].

Criterion for Absence of Shock Wave. Let us determine the pressure p_{**} of the surrounding gas below which not even a weak SW is formed. When the radius of the vaporized material reaches sizes $r \gg (QM_1/m)^{1/2}$ in the process of dispersion the mean free path length of the molecules in the vaporized material becomes much greater than r. Here M_1 is the mass of heated material. At this stage the molecules of vaporized material will disperse without collisions in the form of an expanding layer. This circumstance along with the pulsations of the vaporized material about the equilibrium radius will not be taken into account. For the rest the statement of the problem is analogous to the preceding problem.

The expression for the initial radius of a weak SW

$$R \approx \frac{2k}{\gamma+1}\sqrt{\frac{8}{\pi\gamma}}\left[\frac{4}{3}+\frac{(\gamma-1)(9\gamma-5)}{4\gamma}\right]l \qquad (6)$$

follows from (1) if the expression obtained in [9] is used for L and it is considered that

$$\rho_2/\rho_1 - 1 \approx 4\,(M-1)/(\gamma+1) \text{ when } M - 1 \ll 1$$

where M is the Mach number of the SW.

We can determine the pressure p_{**} from the condition that the initial SW radius corresponding to this pressure equals the radius of the vaporized material which has expanded to the pressure of the surrounding gas.

For this we must find W. It follows from (3) that $W_1 \to 0$ as $M \to 1$. We note that the momentum of the gas in a weak SW front is finally

$$I/S \to \frac{4}{\gamma+1}\sqrt{\frac{2}{\pi\gamma}}\left[\frac{4}{3}+\frac{(\gamma-1)(9\gamma-5)}{4\gamma}\right]Ac_1$$

as $M \to 1$. Thus, for a weak SW, W coincides with the increase in the internal energy of the gas in the volume occupied by the SW front. Setting the average gas pressure in the SW front equal to $(p_2+p_1)/2$ we obtain

$$W/S = (p_2+p_1)\,L/2\,(\gamma+1) - p_1 L/(\gamma-1) - p_1 R/k\,(\gamma-1)$$

If it is considered that

$$p_2/p_1 - 1 \approx 4\gamma\,(M-1)/(\gamma+1) \text{ when } M - 1 \ll 1$$

we arrive at the expression $W/S = p_1R/k$ for the work done by the vaporized material in expanding to the pressure of the surrounding gas.

The equation of conservation of energy takes the form

$$\frac{1}{k}\,p_{**}S(p_{**})\,R(p_{**})\left(\frac{1}{\gamma_1-1}+1\right) = E$$

from which we find with the help of (6) that

$$p_{**} \approx 10\sqrt{\frac{\gamma_1}{\gamma_1-1}}\left\{\frac{2}{\gamma+1}\sqrt{\frac{8}{\pi\gamma}}\left[\frac{4}{3}+\frac{(\gamma-1)(9\gamma-5)}{4\gamma}\right]\right\}^{3/2} B\sqrt{\frac{B}{E}}$$

For example, for air with E =3 J and $\gamma_1 = 5/3$ we obtain $p_{**} \approx 10^{-4}$ mm Hg.

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